R-ratios and moments of nuclear structure functions

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Abstract

We study implications of a model, which links nuclear and nucleon structure functions. For it, computed Callen-Gross functions $\kappa^A(x,Q^2) = 2xF_1^A(x,Q^2)/F_2^A(x,Q^2)$ are for finite Q^2 close to their asymptotic value 1. Using those κ , we compute R ratios for $Q^2 \gtrsim 5 \text{ GeV}^2$. We review approximate methods for the extraction of R from inclusive scattering and EMC data. We also calculate ratios of moments of F_k^A and find these to describe the data and in particular their Q^2 dependence. The above observables, as well as inclusive cross sections, are sensitive tests for the underlying relation between nucleonic and nuclear structure functions. In view of the overall agreement, we speculate that the above relation effectively circumvents a QCD calculation.

In the following we discuss two topics related to nuclear structure functions (SF), namely ratios R^A of cross sections for longitudinal and transverse virtual photons, and ratios of moments of SF. We start with the cross section per nucleon for inclusive scattering of high-energy electrons from nuclei

$$\frac{d^2 \sigma_{eA}(E; \theta, \nu)/A}{d\Omega \, d\nu} = \frac{2}{A} \sigma_M(E; \theta, \nu) \left[\frac{xM^2}{Q^2} F_2^A(x, Q^2) + \tan^2(\theta/2) F_1^A(x, Q^2) \right] \tag{1}$$

The inclusive and the Mott cross section σ_M for point-nucleons are measured as functions of beam energy E, scattering angle θ and energy loss ν . The above nuclear SF $F_k^A(x,Q^2)$ describe the scattering of unpolarized electrons from randomly oriented targets. These depend on the square of the 4-momentum $Q^2 = \mathbf{q}^2 - \nu^2$ and the Bjorken variable x, corresponding to the nucleon mass M with range $0 \leq Q^2/2M\nu \leq A$.

The interest in F_k^A stems from the interplay between nucleonic and sub-nucleonic dynamics which one wishes to study. These are in principle obtained by the Rosenbluth extraction for a single-photon exchange cross section (1), which requires data for fixed x and Q^2 at different scattering angles θ . Since $\sin^2(\theta/2) = Q^2/[4E(E-Q^2/2Mx)]$, varying the scattering angle amounts to varying the beam energy E. Instead of the SF in (1), one extracts the above mentioned ratio R^A [1]

$$R^{A} = d^{2}\sigma_{L}/d^{2}\sigma_{T} = \left(1 + \frac{4M^{2}x^{2}}{Q^{2}}\right) \frac{1}{\kappa^{A}(x, Q^{2})} - 1$$
 (2a)

$$\kappa^{A}(x,Q^{2}) = \frac{2xF_{1}^{A}(x,Q^{2})}{F_{2}^{A}(x,Q^{2})}$$
(2b)

We shall name $\kappa^A(x,Q^2)$ the nuclear Callen-Gross (CG) function.

There exists a rather extensive body of data from which R has been extracted, but the information does not cover wide x, Q^2 ranges and is not accurate, reflecting a similar uncertainty in F_k^A . Below we shall discuss computed results for R and standard approximations.

Eq. (1) holds irrespective of the dynamics underlying the description of the nuclei. With nucleons as dominant degrees of freedom, it is appealing to relate SF of nuclei to those of nucleons, which are considered to be composite for the high Q^2 involved. We shall use below a proposed relation [2]

$$F_k^A(x,Q^2) = \int_x^A \frac{dz}{z^{2-k}} f^{PN}(z,Q^2) F_k^{\langle N \rangle} \left(\frac{x}{z},Q^2\right), \tag{3}$$

where $F_k^{\langle N \rangle}$ are properly p, n-weighted SF's of free nucleons F_k^p , $F_k^n \approx F_k^D/2 - F_k^p$. Those contain information on the sub-structure of the nucleon and we shall use data compiled for F_1^N , and parametrizations for F_2^N [3]. Dynamics enter through the SF of a nucleus with point-particles f^{PN} , probed at high Q^2 . The relation (3) is thought to be valid for both $Q^2 \gtrsim (1-1.5) \text{GeV}^2$ and for $x \gtrsim 0.15$, below which neglected pionic [4] and anti-screening effects grow in importance. In addition, $A \gtrsim 12$ in view of the neglect of nucleon recoil. Applications to cross sections data [5,6] have met with definite success [7,8].

We have demonstrated before that the above f^{PN} is only weakly A-dependent as are the weighted $F_k^{\langle N \rangle}$, even for the largest neutron excess $\delta N/A$. Eq. (3) through (2a) then implies

$$\kappa^{A} = \kappa^{\langle N \rangle} + \mathcal{O}(1/A) \approx \kappa^{D}(x, Q^{2}) + \mathcal{O}(1/A)$$

$$R^{A}(x, Q^{2}) \approx R(x, Q^{2}) + \mathcal{O}(1/A), \tag{4}$$

in agreement with data [1,9]. Using first the CG relation for nucleons

$$\epsilon_{CG}^{N} = \lim_{Q^2 \to \infty} \kappa^{N}(x, Q^2) = 1 \tag{5}$$

one finds from (2b) and (4), its nuclear analog,

$$\epsilon_{CG}^{A} = \lim_{Q^2 \to \infty} \kappa^{A}(x, Q^2) = 1 + \mathcal{O}(1/A) \tag{6}$$

With (5), the nuclear CG relation (6) can be proven directly from (3). In contradistinction, the equality of nuclear and nucleonic CG functions (4) is compatible with (3), but does not necessarily follow from it.

First we mention a remarkable observation for the computed CG functions

$$|\kappa(x, Q^2) - 1| \approx (0.11 - 0.12)$$

 $(0.2 - 0.3) \lesssim x \lesssim (0.7 - 0.75); \ Q^2 \ge 5 \text{GeV}^2$ (7)

In the indicated x-interval and over a wide Q^2 -range, CG functions appear to be close to their asymptotic limit, the nuclear CG relation. It is also intriguing that without any apparent cause, a sign change occurs at a weakly Q^2 -dependent $x_s \approx 0.5 - 0.6$. The above is in agreement with data from high energy $\nu, \bar{\nu}$ inclusive scattering (see Fig. 18 in [10]). The small $\kappa - 1$ shall be shown to entail disproportionally large effects. For later use we remark on the estimated accuracy of the computed CG function (2b), which appears limited in various ranges:

- i) Disregarding other than valence quarks, requires smoothing of F_k^N for $x \lesssim 0.15$ -0.20, which entails the same for F_k^A . We thus prefer to use extrapolated values for nuclear CG functions, below $x \lesssim 0.15$.
- ii) Eq. (3) shows that f^{PN} draws on an ever smaller support of dwindling intensity and accuracy, rendering $F_k^A(x,Q^2)$ unreliable beyond $x \ge 1.3 1.5$.
- iii) The parametrizations for F_2^p , F_2^D [3] hold for $Q^2 \leq 20$ GeV², causing uncertainties in F_k^A for larger Q^2 .
- iv) With SF for $x \geq 1.2$ falling orders of magnitudes from the maximum values, one expects inaccuracies if F_k^A and κ for growing x.

We now discuss three approximations R_n for $R^A \approx R$, defined by a corresponding choice for the CG function κ_n . For each of these one has from (2a)

$$R(x,Q^2) = \beta_n(x,Q^2)R_n(x,Q^2) + \left(\beta_n(x,Q^2) - 1\right)$$
(8)

Deviations of $\beta_n(x, Q^2) = \kappa_n(x, Q^2)/\kappa(x, Q^2)$ from 1 manifestly determine the quality of the approximation.

A) A high- Q^2 approximation, defined by $\kappa_L = 1$ (i.e. $\beta_L = \kappa^{-1}$), approximately valid for $1 \lesssim x \lesssim 0.6$:

$$R^{'exact'}(x,Q^2) = \beta_L(x,Q^2)R_L(x,Q^2) + \left(\beta_L(x,Q^2) - 1\right)$$
(9a)

$$\approx R_L(x, Q^2) + \left(\beta_L(x, Q^2) - 1\right) \tag{9b}$$

$$R_L^{(1)}(x,Q^2) = \frac{4M^2x^2}{Q^2} + (\beta_L(x,Q^2) - 1)$$
(9c)

$$R_L^{(2)}(x,Q^2) = \frac{4M^2x^2}{Q^2} \tag{9d}$$

Eq. (9a) is the same as Eqs. (2a). The corresponding R is dubbed 'exact', because it results from computed values of F_k^A , Eq. (3) [8], which implies some model. $R^{\text{'exact'}}$ should be distinguished from intrinsic approximations for R.

B) The NE approximation for $x \approx 1$ rests on the decomposition of F_k^N in (3) into p, nweighted nucleon-elastic (NE) and nucleon-inelastic (NI) parts. Retention of the NE part
generates through (3) corresponding NE parts in the nuclear SF, thus with $\eta = Q^2/4M^2$

$$F_1^{N(NE)}(x,Q^2) = \frac{x}{2} [G_M^N(Q^2)^2 \delta(x-1)]$$

$$F_2^{N(NE)}(x,Q^2) = \frac{[G_E^N(Q^2)]^2 + \eta [G_M^N(Q^2)]^2}{1+\eta} \delta(x-1)$$

$$F_1^{A(NE)}(x,Q^2) = \frac{1}{2} f^{PN}(x,Q^2) [G_M^N(Q^2)]^2$$

$$F_2^{A(NE)}(x,Q^2) = x f^{PN}(x,Q^2) \frac{[G_E^N(Q^2)]^2 + \eta [G_M^N(Q^2)]^2}{1+\eta}$$
(10b)

The corresponding CG function can be simplified by exploiting the approximate scaling of the static electro-magnetic form factors in the NE part (10a), $1/[(\mu_M^p)^2 + (\mu_M^n)^2] = 0.0874$ [11]

$$\kappa_{NE}^{A} = 2x F_{1}^{A(NE)} / F_{2}^{A(NE)}$$

$$\approx (0.0874 + \eta) / (1 + \eta) \tag{11}$$

Inserting (11) into (3) gives

$$R(x, Q^2) = \beta_{NE}(x, Q^2) R_{NE}(x, Q^2) + \left(\beta_{NE}(x, Q^2) - 1\right)$$
(12a)

$$R_{NE}^{(1)}(x,Q^2) = \frac{0.31}{Q^2} + \left(\frac{0.31}{Q^2} + 1\right) \left(\frac{x^2 - 1}{1 + \eta}\right),\tag{12b}$$

$$R_{NE}^{(2)}(x,Q^2) \approx \frac{0.31}{Q^2},$$
 (12c)

with Q^2 expressed in GeV². Eq. (12c) is the result of Bosted et al [11], while Eq. (12b) provides x-dependent corrections.

C) An empirical estimate for moderate Q^2 , which is assumed to be independent of x and A [11–13]

$$R_C(x, Q^2) \approx \frac{\delta}{Q^2} \quad ; 0.2 \lesssim \delta \lesssim 0.5,$$
 (13)

The estimates (9d), (12c) for $x \approx 1$, and (13) predict $R \propto 1/Q^2$, but only A) and B) for $x \neq 1$ prescribe definite x dependence. Since by definition R depends on x, it is likely that extracted coefficients of $1/Q^2$ effectively hide actual x-dependence.

Were it not for the listed inaccuracies in computed CG functions, the latter would through (2a) or (9a) provide a standard for all approximate R ratios. We now discuss those and start with the large Q^2 approximation. In view of the observation (7), the CG function $\kappa \approx 1$ holds also for moderate Q^2 and over a relatively wide x-range. For 'medium' x^2/Q^2 , which does not require large Q^2 , $R \approx R^{(2)}$ may suffice. However, in the deep-inelastic region for small enough x^2/Q^2 , even for a few % deviation of β_L from 1, the second part in (9c) exceeds $R_L^{(2)}$, and (9c) should therefore be used there.

In Table I we present results for relatively low x, $0.12 \lesssim x \lesssim 0.7$ and for $Q^2 \geq 5$ GeV². The first row gives $R'^{exact'}$, Eq. (9a), computed from (3), except the entry for x = 0.12 which, as explained above, has been extrapolated down from slightly larger x. The second row is the asymptotic limit $R_L^{(2)}$, Eq. (9d). We do not display $R_L^{(1)}$, since it virtually coincides with $R'^{exact'}$. One notices that for higher x, the asymptotic limit is either close to, or exceeds the exact answer. This reflects on κ , Eq. (2b) to be close to, or exceeding 1, in turn entailing a negative correction to $R_L^{(2)}$. This agrees with the observation (7). The last column contains a few scattered ν , $\bar{\nu}$ data for the indicated x and binned $\langle Q \rangle^2$ [14,10]. Given the substantial statistical and systematic errors and the imprecisely given spreading due to binning, the agreement is reasonable.

Next we discuss the NE approximation, the validity of which depends foremost on the weight of $F_k^{N(NE)}(x,Q^2)$ in F_k^A . When using (3), that weight is determined by f^{PN} , for which there is only theoretical information. Computations show that only for $Q^2 \lesssim 2 \text{ GeV}^2$, $F_k^{A(NE)}(x,Q^2)$ dominates for $x \lesssim (1.1\text{-}1.2)$. For growing Q^2 NI parts compete for ever growing x and ultimately overtake [8].

Disregarding NI contributions to R_{NE} for $x \neq 1$, corrections in the immediate neigh-

borhood of the QEP can be estimated by choosing β_{NE} close to 1. One thus finds $R(1.05,5)/R^{NE}(1,5)=1.86$ which ratio rapidly increases with β_{NE} . One also checks from (12b) that for $1 \lesssim Q^2(\text{GeV}^2) \lesssim 5$, $R_{NE}(x \lesssim 0.9, Q^2)$ reaches unphysical negative values. Only the disregarded NI part can restore R to positive values. For $1.5 \gtrsim Q^2(\text{GeV}^2) \gtrsim 5$ and for instance x=1.1 on the elastic side of the QEP, $2 \gtrsim R_{NE}(1.1,Q^2)/R_{NE}(1,Q^2) \gtrsim 1.5$, which ratio again grows with x: NI terms may, or may not off-set that growth. Table II compares the NE approximations $R_{NE}^{(1)}$, $R_{NE}^{(2)}$ with R_C : the agreement is tolerable. Aware of the warnings after (7), we nevertheless compute and enter some 'exact' values, which appear to exceed the NE values by far. CG functions $\kappa(1,Q^2)$ which fit R_{NE} would have to be 25-30 % larger than the computed ones, which we estimate to be outside the limits of our accuracy. In particular the negative $R_{NE}(0.9,Q^2)$ makes one believe that the NE estimates may not be precise.

Eq. (12c) has been applied to extract R and F_2^A from inclusive scattering data for medium- Q^2 data for $x \approx 1$ [13,15]. Data by Bosted et al for $0.75 \lesssim x \lesssim 1.15$ are quite eratic, but $R(\langle x \rangle, Q^2)$, averaged over x, shows a trend in agreement with (12c).

In addition there are data for about the same Q^2 -range, but more restricted x [15], which are in agreement with either (12c) or (13). There clearly are substantial corrections just off the QEP. In particular for the data of Bosted et al, the above warns that the use of simple x-independent R ratios may lead to extracted F_2^A , which have inaccuracies, exceeding those estimated.

We now address a second topic regarding the moments of various SF

$$\mathcal{M}_{k}^{A}(m;Q^{2}) = \int_{0}^{A} dx x^{m} F_{k}^{A}(x,Q^{2})$$

$$\mathcal{M}_{k}^{N}(m;Q^{2}) = \int_{0}^{1} dx x^{m} F_{k}^{N}(x,Q^{2})$$

$$\mu^{A}(m;Q^{2}) = \int_{0}^{A} dx x^{m} f^{PN}(x,Q^{2})$$
(14)

Moments \mathcal{M}_k^N describe higher twist corrections of SF of nucleons [16], and the same holds for their nuclear counterparts, had those been calculated in QCD. Our interest in those

moments is the sensitivity of SF for large x and consequently the trust in the calculated F_k^A for that range. One readily derives from (10) [17]

$$F_k^A(0, Q^2) = \mu^A(-2 + k; Q^2)F_k^N(0, Q^2)$$
(15a)

$$\mathcal{M}_{k}^{A}(m,Q^{2}) = \mu^{A}(m-1+k;Q^{2})\mathcal{M}_{k}^{N}(m;Q^{2})$$
(15b)

$$\mu^{A}(m+1;Q^{2}) = \frac{\mathcal{M}_{1}^{A}(m+1;Q^{2})}{\mathcal{M}_{1}^{N}(m+1;Q^{2})} = \frac{\mathcal{M}_{2}^{A}(m;Q^{2})}{\mathcal{M}_{2}^{N}(m;Q^{2})}$$
(15c)

and in particular

$$\mu^{A}(0,Q^{2}) = \int_{0}^{A} dx f^{PN}(x,Q^{2}) = \int_{0}^{A} dx f^{as}(x) = 1, \tag{16}$$

which expresses unitarity. All other relations (15) for finite Q^2 rest on the representation (3) and embody effects of the binding medium on moments of F_k^N through $\mu(n, Q^2)$. For instance, the deviation of $\mu^A(2, Q^2)$ from 1 measures the difference of the momentum fraction of a quark in a nucleus and in the nucleon at given Q^2 .

We have computed the lowest moments and ratios μ from computed F_k^A , f^{PN} and parametrized F_k^N . With expected inaccuracies in F_k^A for $x \gtrsim 1.5$ one ought not to trust calculated higher moments. Yet we found consistent values for the different ratios in (15c) for $Q^{\leq}20~{\rm GeV^2}$, and the moments of f^{PN} . Those for Fe are entered in Fig. 1 and agree reasonably well with the available data. We note in particular the rendition of the observed Q^2 -dependence, as opposed to a similar investigation by Cothran et al [18]. The authors used a generalized convolution like (3), with a Q^2 -independent PWIA for f^{PN} , leading to the same for $\mu(m)$. Q^2 -dependence, estimated for off-shell nucleons, produce far too small moment ratios with the wrong Q^2 behavior.

The above is reminiscent of previously considered, but not identical moments. We recall discrepancies between data and computed results for relatively low-q, longitudinal responses S_L and the integral of the latter, the Coulomb sumrule [19,20]. All have occasionally been ascribed to the influence of the binding medium on the size of a nucleon, i.e. on the second moment of the static charge density. Apart from possible conventional accounts of those

differences [21], one notes that (3) does not relate to static moments of charge distributions, but to dynamical SF.

The above and Refs. [7,8] conclude a program to determine observables which depend on nuclear SF, in turn computed from the basic relation (2a) between SF for composite nuclei, free nucleons and of a nucleus composed of point nucleons. The various observables occasionally extend over wide ranges, and test to various measures the x, Q^2 dependence of F_k^A . It is gratifying to frequently note good agreement with data.

The above clearly requires an explanation, because results have been obtained, circumventing QCD. It seems hard to avoid the conclusion that in the tested x, Q^2 region, the relation (2a) is result of an effective theory, as has been argued originally [2] and somehow mimicking notions of QCD.

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Table I

	$Q^2({ m GeV^2})$	5	10	20	50		
x	R						
	$R^{'exact'}$	0.284	0.226	0.221	0.218		
0.08	$R_L^{(2)}$	0.005	0.002	0.001	0.000		
	$R^{exp}(\langle Q^2 \rangle \approx 7)$	$0.27 \pm$	0.06 ± 0.02				
	$R^{'exact'}$	0.216	0.203	0.185	0.176		
0.12	$R_L^{(2)}$	0.010	0.005	0.003	0.001		
	$R^{exp}(\langle Q^2 \rangle \approx 12)$						
	$R^{'exact'}$	0.192	0.169	0.146	0.120		
0.18	$R_L^{(2)}$	0.023	0.011	0.005	0.002		
	$R^{exp}(\langle Q^2 \rangle) \approx 23)$			C	$0.06 \pm 0.06 \pm 0.02$		
	$R'^{exact'}$	0.159	0.120	0.089	0.044		
0.27	$R_L^{(2)}$	0.051	0.025	0.013	0.006		
	$R^{exp}(\langle Q^2 \rangle \approx 30)$			$0.04 \pm 0.04 \pm 0.01$			
	$R^{'exact'}$	0.144	0.119	0.064	0.009		
0.36	$R_L^{(2)}$	0.091	0.025	0.013	0.006		
	$R^{exp}(\langle Q^2 \rangle \approx 50)$				$-0.04 \pm 0.04 \pm 0.01$		
0.5	$R^{'exact'}$	0.140	0.113	0.048	≈ 0		
	$R_L^{(2)}$	0.178	0.089	0.044	0.018		
0.7	$R^{'exact'}$	0.223	0.170	0.120	≈ 0		
	$R_L^{(2)}$	0.348	0.170	0.085	0.035		

'Exact' R for low x and medium-high Q^2 , the high Q^2 limit and data for binned $\langle Q^2 \rangle$ [14,10]. The first row for x = 0.1 are extrapolations down to x = 0.1.

Table II

x	R	$Q^2(\text{GeV}^2)$: 2	5	10
0.9	$R_{NE}^{(1)}$	< 0	< 0	< 0
	$R_{NE}^{(2)}$	0.155	0.062	0.032
	$R^{'exact'}$	-	0.292	0.308
1.0	$R_{NE}^{(1)}$	0.155	0.062	0.032
	$R_{NE}^{(2)}$	0.155	0.062	0.032
	$R^{'exact'}$	-	0.329	0.404
1.05	$R_{NE}^{(1)}$	0.231	0.117	0.059
	$R_{NE}^{(2)}$	0.155	0.062	0.031
x	$R_C([0.4 \le \delta \le 0.6])$	0.2-0.3	0.08-0.12	0.04-0.06

R ratios (12b), (12c) for $x\approx 1$, medium- Q^2 and the x-independent R_c , Eq. (13). For x=0.9,1.0; $Q^2=5,10$ GeV² we also entered $R^{'exact'}(x,Q^2)$. See text for discussion.

figure captions.

Fig. 1 Second, third and fourth moments $\mu(m,Q^2)$, Eq. (16).

